



HILLSBOROUGH TOWNSHIP PUBLIC SCHOOLS

Office of Curriculum and Instruction

CURRICULUM MAP

COURSE TITLE	AP Precalculus							
GRADE BAND		PreK-4		5-6		7-8	<input checked="" type="checkbox"/>	9-12
DEPARTMENT	Mathematics							
LAST REVISION DATE	N/A							
BOE APPROVAL DATE	5/22/2023							

COURSE OVERVIEW

AP Precalculus is a full year, rigorous course in which students study a broad spectrum of function types that are foundational for careers in mathematics, physics, biology, health science, social science, and data science. AP Precalculus prepares students for other college-level mathematics and science courses. Through regular practice, students build deep mastery of modeling and functions, and they examine scenarios through multiple representations.

Prerequisites: Students need to have a strong foundation in Algebra I, Geometry, and Algebra II providing a proficiency in linear and quadratic functions, systems of equations, and familiarity with radical, exponential, and trigonometric expressions.

Pedagogical Approach: The AP Precalculus curriculum employs a standards-based learning model. It takes a student-centered approach to help realize the goal of high achievement for all students. The course promotes student engagement, independent thought and interactive collaboration with peers. Student-centric lessons, activities and assessments are paired with augmentative teacher-centric lessons, activity and task guides, and reporting to empower teachers to empower students.

Three Mathematical Thinking Practices: There are eight distinct skills associated with three mathematical practices. Students should build and master these skills throughout the course to deepen understanding and build bridges across topics. These practices can be assessed in all topics.

Hillsborough Township Public Schools Curriculum Map
Course Title: AP Precalculus

Mathematical Thinking Practices		
Practice 1	Practice 2	Practice 3
Procedural and Symbolic Fluency	Multiple Representations	Communication and Reasoning

Five Main Units: The course material focuses on four big branches of mathematics. The first three are tested on the AP Exam. The last unit contains an extension past the AP College Board Curriculum but is a necessary bridge to calculus courses.

Mathematical Units			
Unit 1 (PRF)	Unit 2 (ELF)	Unit 3 (TPF)	Unit 4 (PVM)
Polynomial and Rational Functions	Exponential and Logarithmic Functions	Trigonometric and Polar Functions	Functions Involving Parameters, Vectors, and Matrices; Intro to Limits

Hillsborough Township Public Schools Curriculum Map
Course Title: AP Precalculus

UNIT OF STUDY	Unit 1: Polynomial and Rational Functions
PACING	53 days
ESSENTIAL QUESTIONS	
<ul style="list-style-type: none">● How do we model the intensity of light from its source?● How can I use data and graphs to figure out the best time to purchase event tickets?● How can we adjust known projectile motion models to account for changes in conditions?	
ENDURING UNDERSTANDINGS	
<ul style="list-style-type: none">● Output values change in tandem with changing input values which is also called covariation.● Rates of change including average rate of change, rate of change at a point, and changing rates of change can help make better estimates.● A function is a rule for relating two simultaneously changing sets of values which provides a vital tool to take into account changes such as crop yield increasing but at a decreasing rate or the efficacy of a medicine decreasing but at an increasing rate.	
LEARNING TARGETS	
<ul style="list-style-type: none">● Chapter 1: Functions and Their Properties<ul style="list-style-type: none">○ 1.1 Modeling and Equation Solving<ul style="list-style-type: none">■ Students will be able to use data to create a mathematical model.■ Students will be able to use algebraic models to determine unknown quantities by relating to known quantities.■ Students will be able to use graphical models to create visual representations of data, for example: fitting a curve to data.○ 1.2 Function Behavior<ul style="list-style-type: none">■ Students will be able to determine if a relation is a function both algebraically and graphically.■ Students will be able to determine the domain, range, and continuity of a function.■ Students will be able to determine increasing and decreasing intervals, identify local and absolute extrema, and identify concavity and points of inflection.■ Students will be able to determine even and odd symmetry of a relation.○ 1.3 Twelve Basic Functions<ul style="list-style-type: none">■ Students will be able to graph linear, quadratic, cubic, square root, logarithm, reciprocal, exponential, sine, cosine, greatest integer, absolute value, and cube root functions as well as determine function behavior.○ 1.4 Building Functions from Functions	

Hillsborough Township Public Schools Curriculum Map

Course Title: AP Precalculus

- Students will be able to combine functions algebraically and find the sum, difference, product and quotient of functions.
- Students will be able to compose functions and find the domain of a composition.
- Students will be able to decompose a function.
- Students will be able to use implicitly defined relations.
- 1.5 Parametric Functions and Inverse Functions
 - Students will be able to define functions parametrically.
 - Students will be able to find inverse relations and inverse functions both algebraically and graphically.
- 1.6 Graphical Transformations
 - Students will be able to complete horizontal and vertical translations algebraically and graphically.
 - Students will be able to reflect over x-axis and y-axis algebraically and graphically.
 - Students will be able to compose functions with absolute value algebraically and graphically.
 - Students will be able to complete horizontal and vertical stretches and shrinks algebraically and graphically.
- 1.7 Modeling with Functions
 - Students will be able to create multiple representations of the same data using different dependent variables.
 - Students will be able to create functions from graphs and verbal descriptions.
 - Students will be able to optimize quadratic functions.
 - Students will be able to convert units.
 - Students will be able to find the regression line using technology.
- Chapter 2: Polynomial and Rational Functions
 - 2.1 Function Families, Linear Functions, and Linear Models
 - Students will be able to determine the degree of a function and classify by degree.
 - Students will be able to model linear data and find regression lines.
 - Students will be able to find the average rate of change and its relation to slope.
 - 2.2 Quadratic Functions and Modeling
 - Students will be able to model quadratic data graphically and algebraically and find regression lines.
 - Students will be able to find the average rate of change for various points on a quadratic function.
 - Students will be able to determine if the slope is increasing or decreasing on a quadratic function or its concavity.
 - 2.3 Polynomial Functions and Modeling
 - Students will be able to graph polynomial functions by finding points of inflection, concavity, end behavior, and zeros.

Hillsborough Township Public Schools Curriculum Map

Course Title: AP Precalculus

- Students will be able to use Intermediate Value Theorem to approximate zeros.
 - Students will be able to model polynomial functions algebraically.
- 2.4 Real Zeros of Polynomial Functions
 - Students will be able to divide polynomial functions using long division and synthetic division.
 - Students will be able to use the Remainder and Factor Theorems.
 - Students will be able to find the rational and real zeros of a polynomial function using Rational Zeros Theorem.
 - Students will be able to use the Binomial Theorem.
- 2.5 Complex Zeros and the Fundamental Theorem of Algebra
 - Students will be able to use the Fundamental Theorem of Algebra, Linear Factorization Theorem, and Complex Conjugate Zeros Theorem to find complex zeros.
 - Students will be able to factor polynomials such that it has real coefficients.
 - Students will be able to use the complex plane to graph complex zeros.
- 2.6 Rational Functions and Their Graphs
 - Students will be able to find domain, range, end behavior, x-intercept(s), y-intercept, and holes.
 - Students will be able to find linear and non-linear asymptotes and describe in limit notation.
 - Students will be able to graph and analyze a rational function.
- 2.7 Rational Equations, Inequalities, and Modeling
 - Students will be able to solve polynomial equations and inequalities.
 - Students will be able to model a rational function and solve graphically or using technology.
 - Students will be able to identify extraneous solutions.

ASSESSMENTS

Pre-Assessment(s)	<ul style="list-style-type: none"> ● Summer Work Multiple Choice Assessment
Formative	<ul style="list-style-type: none"> ● Homework ● Exit Tickets ● AP Classroom Problems
Summative	<ul style="list-style-type: none"> ● Standard Mapped Quizzes ● Challenge Problems ● Bottle Lab
Benchmark / Common	<ul style="list-style-type: none"> ● Quarter 1 Exam

COLLEGE BOARD AP PRECALCULUS LEARNING STANDARDS **(CBAPPS)**

Must include the standard # & verbiage

Hillsborough Township Public Schools Curriculum Map
Course Title: AP Precalculus

- 1.1.A: Describe how the input and output values of a function vary together by comparing function values.
- 1.1.B: Construct a graph representing two quantities that vary with respect to each other in a contextual scenario.
- 1.2.A: Compare the rates of change at two points using average rates of change near the points.
- 1.2.B: Describe how two quantities vary together at different points and over different intervals of a function.
- 1.3.A: Determine the average rates of change for sequences and functions, including linear, quadratic, and other function types.
- 1.3.B: Determine the change in the average rates of change for linear, quadratic, and other function types.
- 1.4: Identify key characteristics of polynomial functions related to rates of change.
- 1.5.A: Identify key characteristics of a polynomial function related to its zeros when suitable factorizations are available or with technology.
- 1.5.B: Determine if a polynomial function is even or odd.
- 1.6: Describe end behaviors of polynomial functions.
- 1.7: Describe end behaviors of rational functions.
- 1.8: Determine the zeros of rational functions.
- 1.9: Determine vertical asymptotes of graphs of rational functions.
- 1.10: Determine holes in graphs of rational functions.
- 1.11.A: Rewrite polynomial and rational expressions in equivalent forms.
- 1.11.B: Determine the quotient of two polynomial functions using long division.
- 1.11.C: Rewrite the repeated product of binomials using the binomial theorem.
- 1.12: Construct a function that is an additive and/or multiplicative transformation of another function.
- 1.13.A: Identify an appropriate function type to construct a function model for a given scenario.
- 1.13.B: Describe assumptions and restrictions related to building a function model.
- 1.14.A: Construct a linear, quadratic, cubic, quartic, polynomial of degree n , or related piecewise-defined function model.
- 1.14.B: Construct a rational function model based on a context.
- 1.14.C: Apply a function model to answer questions about a data set or contextual scenario.
- 2.6.A: Construct linear, quadratic, and exponential models based on a data set.
- 2.6.B: Validate a model constructed from a data set.
- 2.7.A: Evaluate the composition of two or more functions for given values.
- 2.7.B: Construct a representation of the composition of two or more functions.
- 2.7.C: Rewrite a given function as a composition of two or more functions.
- 2.8.A: Determine the input-output pairs of the inverse of a function.
- 2.8.B: Determine the inverse of a function on an invertible domain.
- 4.1: Construct a graph or table of values for a parametric function represented analytically.
- 4.5.B: Determine how the two quantities related in an implicitly defined function vary together.

Hillsborough Township Public Schools Curriculum Map
Course Title: AP Precalculus

INTERDISCIPLINARY CONNECTIONS

NEW JERSEY STUDENT LEARNING STANDARDS (NJSL)

Must include the standard # & verbiage

Comprehensive Health & Physical Education

- 2.1.12.EH.1: Recognize one's personal traits, strengths, and limitations and identify how to develop skills to support a healthy lifestyle.
- 2.1.12.EH.3: Describe strategies to appropriately respond to stressors in a variety of situations.

English Language Arts

- RST.11-12.4 - Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context.
- W.11-12.1. Write arguments to support claims in an analysis of substantive topics or texts, using valid reasoning and relevant and sufficient evidence.

Computer Science & Design Thinking

- 8.1.2.CS.1: Select and operate computing devices that perform a variety of tasks accurately and quickly based on user needs and preferences.
- 8.1.2.DA.1: Collect and present data, including climate change data, in various visual formats.
- 8.1.2.DA.3: Identify and describe patterns in data visualizations.
- 8.1.2.DA.4: Make predictions based on data using charts or graphs.
- 8.1.2.AP.4: Break down a task into a sequence of steps.
- 8.1.5.DA.5: Propose cause and effect relationships, predict outcomes, or communicate ideas using data.

Science

- None.

World Languages

- None.

CAREER READINESS, LIFE LITERACIES, & KEY SKILLS CONNECTIONS

Must include the standard # & verbiage

9.1-Personal Financial Literacy

- None.

9.2-Career Awareness, Exploration, Preparation, and Training

Hillsborough Township Public Schools Curriculum Map
Course Title: AP Precalculus

- 9.4.12.CI.1: Demonstrate the ability to reflect, analyze, and use creative skills and ideas.
- 9.4.5.DC.4: Model safe, legal, and ethical behavior when using online or offline technology.

9.4-Life Literacies & Key Skills

- 9.4.12.TL.3: Analyze the effectiveness of the process and quality of collaborative environments.
- 9.4.12.CT.2: Explain the potential benefits of collaborating to enhance critical thinking and problem solving.
- 8.2.2.ED.2: Collaborate to solve a simple problem, or to illustrate how to build a product using the design process.

CAREERS ASSOCIATED WITH THIS UNIT

- Data analyst, physicist

DIVERSITY, EQUITY, & INCLUSION CONNECTIONS

*Required in grades K-12 per [N.J.S.A. 18A:35-4:36a](#) & the Amistad Law [N.J.S.A. 18A 52:16A-88](#)
Required in grades 7-12 per [N.J.S.A. 18A:35-4.35](#)*

- None.

SOCIAL EMOTIONAL LEARNING CONNECTIONS

[NJ SEL](#) sub-competencies are listed that are addressed in this unit

Self-Awareness

- Recognize one’s personal traits, strengths, and limitations
- Recognize the importance of self-confidence in handling daily tasks and challenges

Self-Management

- Understand and practice strategies for managing one’s own emotions, thoughts, and behaviors
- Recognize the skills needed to establish and achieve personal and educational goals
- Identify and apply ways to persevere or overcome barriers through alternative methods to achieve one’s goals

Social Awareness

- Recognize and identify the thoughts, feelings, and perspectives of others
- Demonstrate an awareness of the expectations for social interactions in a variety of settings

Relationship Skills

- Utilize positive communication and social skills to interact effectively with others

Hillsborough Township Public Schools Curriculum Map
Course Title: AP Precalculus

- Identify who, when, where, or how to seek help for oneself or others when needed

Responsible Decision-Making

- Develop, implement, and model effective problem-solving and critical thinking skills
- Identify the consequences associated with one's actions in order to make constructive choices

MODIFICATIONS/ACCOMMODATIONS - *ELL, Special Education, Gifted, At Risk of Failure, 504*

- Accommodations for all subject areas may be viewed [here](#).

RESOURCES – *Cited print and electronic sources*

- AP Classroom
- College Board
- Calc Medic
- Desmos
- GeoGebra
- Classpad.net (Casio)
- TI Education (Texas Instruments)

Hillsborough Township Public Schools Curriculum Map
Course Title: AP Precalculus

UNIT OF STUDY	Unit 2: Exponential and Logarithmic Functions
PACING	19 days
ESSENTIAL QUESTIONS	
<ul style="list-style-type: none">● How can I make a single model that merges the interest I earn from my bank with the taxes that are due so I can know how much I will have in the end?● How can we adjust the scale of distance for a model of planets in the solar system so the relationships among the planets are easier to see?● If different functions can be used to model data, how do we pick which one is best?	
ENDURING UNDERSTANDINGS	
<ul style="list-style-type: none">● When an aspect of a phenomenon changes proportionally to the existing amount, exponential and logarithmic models are employed to harness the information.● Exponential functions are key to modeling population growth, radioactive decay, interest rates, and the amount of medication in a patient.● Logarithmic functions are useful in modeling sound intensity and frequency, the magnitude of earthquakes, the pH scale in chemistry, and the working memory in humans.	
LEARNING TARGETS	
<ul style="list-style-type: none">● Chapter 3: Exponential and Logarithmic Functions<ul style="list-style-type: none">○ 3.1 Arithmetic and Geometric Sequences<ul style="list-style-type: none">■ Students will be able to use arithmetic and geometric sequences to find terms in a sequence and common difference or ratio.■ Students will graph sequences to determine linear or exponential and growth or decay.■ Students will be able to use summation notation.■ Students will be able to determine if the sum of an infinite series converges, and find the sum if possible.○ 3.2 Exponential Functions<ul style="list-style-type: none">■ Students will be able to graph and analyze exponential functions.■ Students will be able to use the natural base e.○ 3.3 Exponential Modeling<ul style="list-style-type: none">■ Students will be able to find growth and decay rates to create exponential functions.■ Students will be able to use exponential growth and decay models to solve.■ Students will be able to decide which model best fits a data set and validate with reasoning.○ 3.4 Logarithmic Functions<ul style="list-style-type: none">■ Students will be able to find inverses of exponential functions.■ Students will be able to use common and natural logarithms.	

Hillsborough Township Public Schools Curriculum Map

Course Title: AP Precalculus

- Students will be able to graph and analyze logarithmic functions.
 - 3.5 Properties of Logarithmic Functions
 - Students will be able to use product, quotient, and power rule properties of logarithms to expand and condense logarithmic expressions.
 - Students will be able to use change-of-base formulas for logarithms.
 - 3.6 Logarithmic Scaling and Semi-Log Plots
 - Students will be able to use logarithmic scaling to linearize exponential data.
 - Students will be able to use semi-log plots to determine if an exponential model is appropriate.
 - 3.7 Equation Solving and Modeling
 - Students will be able to solve exponential and logarithmic equations.
 - Students will be able to solve exponential and logarithmic inequalities.
 - Students will be able to use models to create logarithmic functions and use them to solve.
 - Students will be able to use orders of magnitude in order to compare values (ex: comparing earthquake intensities).

ASSESSMENTS

Pre-Assessment(s)	<ul style="list-style-type: none"> ● None
Formative	<ul style="list-style-type: none"> ● Homework ● Exit Tickets ● AP Classroom Problems
Summative	<ul style="list-style-type: none"> ● Standards Mapped Quizzes ● Challenge Questions
Benchmark / Common	<ul style="list-style-type: none"> ● Quarter 2 Exam

COLLEGE BOARD AP PRECALCULUS LEARNING STANDARDS (**CBAPPS**)

Must include the standard # & verbiage

- 2.1.A: Express arithmetic sequences found in mathematical and contextual scenarios as functions of the whole numbers.
- 2.1.B: Express geometric sequences found in mathematical and contextual scenarios as functions of the whole numbers.
- 2.2.A: Construct functions of the real numbers that are comparable to arithmetic and geometric sequences.
- 2.2.B: Describe similarities and differences between linear and exponential functions.
- 2.3: Identify key characteristics of exponential functions.
- 2.4: Rewrite exponential expressions in equivalent forms.
- 2.5.A: Construct a model for situations involving proportional output values over equal-length input-value intervals.

Hillsborough Township Public Schools Curriculum Map
Course Title: AP Precalculus

- 2.5.B: Apply exponential models to answer questions about a data set or contextual scenario.
- 2.6.A: Construct linear, quadratic, and exponential models based on a data set.
- 2.6.B: Validate a model constructed from a data set.
- 2.9: Evaluate logarithmic expressions.
- 2.10: Construct representations of the inverse of an exponential function with an initial value of 1.
- 2.11: Identify key characteristics of logarithmic functions.
- 2.12: Rewrite logarithmic expressions in equivalent forms.
- 2.13.A: Solve exponential and logarithmic equations and inequalities.
- 2.13.B: Construct the inverse function for exponential and logarithmic functions.
- 2.14: Construct a logarithmic function model.
- 2.15.A: Determine if an exponential model is appropriate by examining a semi-log plot of a data set.
- 2.15.B: Construct the linearization of exponential data.

INTERDISCIPLINARY CONNECTIONS

NEW JERSEY STUDENT LEARNING STANDARDS (NJSL)

Must include the standard # & verbiage

Comprehensive Health & Physical Education

- 2.1.12.EH.1: Recognize one's personal traits, strengths, and limitations and identify how to develop skills to support a healthy lifestyle.
- 2.1.12.EH.3: Describe strategies to appropriately respond to stressors in a variety of situations.

English Language Arts

- RST.11-12.4 - Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context.
- W.11-12.1. Write arguments to support claims in an analysis of substantive topics or texts, using valid reasoning and relevant and sufficient evidence.

Computer Science & Design Thinking

- 8.1.2.CS.1: Select and operate computing devices that perform a variety of tasks accurately and quickly based on user needs and preferences.
- 8.1.2.DA.1: Collect and present data, including climate change data, in various visual formats.
- 8.1.2.DA.3: Identify and describe patterns in data visualizations.
- 8.1.2.DA.4: Make predictions based on data using charts or graphs.
- 8.1.2.AP.4: Break down a task into a sequence of steps.
- 8.1.5.DA.5: Propose cause and effect relationships, predict outcomes, or communicate ideas using data.

Hillsborough Township Public Schools Curriculum Map
Course Title: AP Precalculus

Science
<ul style="list-style-type: none"> ● HS-LS2-1: Use mathematical and/or computational representations to support explanations of factors that affect carrying capacity of ecosystems at different scales.
Visual & Performing Arts
<ul style="list-style-type: none"> ● None.
World Languages
<ul style="list-style-type: none"> ● None.
CAREER READINESS, LIFE LITERACIES, & KEY SKILLS CONNECTIONS <i>Must include the standard # & verbiage</i>
9.1-Personal Financial Literacy
<ul style="list-style-type: none"> ● None.
9.2-Career Awareness, Exploration, Preparation, and Training
<ul style="list-style-type: none"> ● 9.4.12.Cl.1: Demonstrate the ability to reflect, analyze, and use creative skills and ideas. ● 9.4.5.DC.4: Model safe, legal, and ethical behavior when using online or offline technology.
9.4-Life Literacies & Key Skills
<ul style="list-style-type: none"> ● 9.4.12.TL.3: Analyze the effectiveness of the process and quality of collaborative environments. ● 9.4.12.CT.2: Explain the potential benefits of collaborating to enhance critical thinking and problem solving. ● 8.2.2.ED.2: Collaborate to solve a simple problem, or to illustrate how to build a product using the design process.
CAREERS ASSOCIATED WITH THIS UNIT
<ul style="list-style-type: none"> ● Biologist, chemist, musician, finance
DIVERSITY, EQUITY, & INCLUSION CONNECTIONS <i>Required in grades K-12 per N.J.S.A. 18A:35-4:36a & the Amistad Law N.J.S.A. 18A 52:16A-88 Required in grades 7-12 per N.J.S.A. 18A:35-4.35</i>
<ul style="list-style-type: none"> ● None.
SOCIAL EMOTIONAL LEARNING CONNECTIONS <i>NJ SEL sub-competencies are listed that are addressed in this unit</i>
Self-Awareness

Hillsborough Township Public Schools Curriculum Map
Course Title: AP Precalculus

- Recognize one’s personal traits, strengths, and limitations
- Recognize the importance of self-confidence in handling daily tasks and challenges

Self-Management

- Understand and practice strategies for managing one’s own emotions, thoughts, and behaviors
- Recognize the skills needed to establish and achieve personal and educational goals
- Identify and apply ways to persevere or overcome barriers through alternative methods to achieve one’s goals

Social Awareness

- Recognize and identify the thoughts, feelings, and perspectives of others
- Demonstrate an awareness of the expectations for social interactions in a variety of settings

Relationship Skills

- Utilize positive communication and social skills to interact effectively with others
- Identify who, when, where, or how to seek help for oneself or others when needed

Responsible Decision-Making

- Develop, implement, and model effective problem-solving and critical thinking skills
- Identify the consequences associated with one’s actions in order to make constructive choices

MODIFICATIONS/ACCOMMODATIONS - ELL, Special Education, Gifted, At Risk of Failure, 504

- Accommodations for all subject areas may be viewed [here](#).

RESOURCES – *Cited print and electronic sources*

- AP Classroom
- College Board
- Calc Medic
- Desmos
- GeoGebra
- Classpad.net (Casio)
- TI Education (Texas Instruments)

Hillsborough Township Public Schools Curriculum Map
Course Title: AP Precalculus

UNIT OF STUDY	Unit 3: Trigonometric and Polar Functions
PACING	57 days
ESSENTIAL QUESTIONS	
<ul style="list-style-type: none">● Since energy usage goes up and down through the year, how can I use trends in data to predict my monthly electricity bills when I get my first apartment?● How do we model aspects of circular and spinning objects without using complex equations from the x-y rectangular-based coordinate system?● How does right triangle trigonometry from geometry relate to trigonometric functions?	
ENDURING UNDERSTANDINGS	
<ul style="list-style-type: none">● Since output values repeat with every full revolution around the circle, trigonometric functions are ideal for modeling periodic, or repeated pattern phenomena such as: tidal waves, blood pressure produced by a heart, moving parts of an analog clock, traffic flow at an intersection over the course of a week, and demand for a particular product.● Polar functions use circular concepts of radii and angles to describe location instead of rectangular concepts of left-right and up-down.● Trigonometry bridges the rectangular and polar coordinate systems by using right triangle trigonometry.	
LEARNING TARGETS	
<ul style="list-style-type: none">● Chapter 4: Trigonometric Functions<ul style="list-style-type: none">○ 4.1 Angles and Their Measure<ul style="list-style-type: none">■ Students will be able to convert between degrees and radians.■ Students will be able to determine arc length of a sector in both degrees and radians.■ Students will be able to use angular speed to find linear speed.○ 4.2 Trigonometry of Acute Angles<ul style="list-style-type: none">■ Students will be able to graph angles in standard position.■ Students will be able to evaluate sine, cosine, tangent, cosecant, secant, and cotangent for acute angles in a right triangle, given sides of the triangle.■ Students will be able to find sine, cosine, tangent, cosecant, secant, and cotangent for 30 degrees, 45 degrees, and 60 degrees.■ Students will be able to find the remaining 5 trigonometric functions when given one trigonometric function.■ Students will be able to solve a right triangle and apply to situational modeling.○ 4.3 The Trigonometric Functions Extended<ul style="list-style-type: none">■ Students will be able to find coterminal angles.	

Hillsborough Township Public Schools Curriculum Map

Course Title: AP Precalculus

- Students will be able to evaluate sine, cosine, tangent, cosecant, secant, and cotangent for non-acute angles.
- Students will be able to find x and y coordinates for angles in the unit circle.
- Students will be able to determine if a function is periodic.
- 4.4 Graphs of Sine and Cosine Sinusoids
 - Students will be able to graph and analyze sine and cosine functions.
 - Students will be able to find amplitude, period, and frequency of a sine and cosine function.
 - Students will be able to model periodic behavior with sinusoids.
- 4.5 Tangent, Cotangent, Secant, and Cosecant
 - Students will be able to graph and analyze tangent, cotangent, secant, and cosecant functions.
- 4.6 Graphs of Composite Trigonometric Functions
 - Students will be able to explore combining trigonometric and algebraic functions.
 - Students will be able to explore sums of sinusoids.
- 4.7 Inverse Trigonometric Functions
 - Students will be able to determine the domain and range for inverse sine, cosine, and tangent functions.
 - Students will be able to use inverse trigonometry to solve for angles.
 - Students will be able to compose trigonometric functions with inverse functions.
 - Students will be able to use inverse trigonometry to solve applications.
- Chapter 5: Equivalent Trigonometric Representations and Polar Functions
 - 5.1 Fundamental Identities
 - Students will be familiar with reciprocal identities, pythagorean identities, cofunction identities, and odd-even identities.
 - Students will be able to simplify trigonometric expressions using identities.
 - Students will be able to solve trigonometric equations using identities.
 - 5.2 Proving Trigonometric Identities
 - Students will be able to prove trigonometric equations using identities.
 - Students will be able to disprove false trigonometric equations graphically or through counter-example.
 - 5.3 Sum and Difference Identities
 - Students will be able to use cosine/sine/tangent sum and difference angle formulas to simplify expressions and find exact values of angles.
 - Students will be able to use cosine/sine/tangent double angle formulas to simplify expressions and find exact values of angles.
 - Students will be able to prove trigonometric equations using sum/difference/double identities.
 - Students will be able to solve trigonometric equations using sum/difference/double identities.
 - 5.4 Laws of Sines and Cosines

Hillsborough Township Public Schools Curriculum Map

Course Title: AP Precalculus

- Students will be able to solve for missing sides and angles in oblique triangles using Law of Cosines or Law of Sines.
- Students will be able to determine the number of triangles that can be created when given SSA.
- Students will be able to find the area of an oblique triangle.
- Students will be able to use Law of Cosines, Law of Sines, and the area formulas to solve triangle applications.
- 5.5 Polar Coordinates and the Complex Plane
 - Students will be able to convert between rectangular and polar coordinates.
 - Students will be able to write multiple polar representations for the same point.
 - Students will be able to convert equations between rectangular and polar form.
 - Students will be able to find distance using polar coordinates.
 - Students will be able to write complex numbers in polar form.
 - Students will be able to multiply and divide polar complex numbers.
 - Students will be able to do powers and roots of polar complex numbers using De Moivre's Theorem.
- 5.6 Polar Functions and Their Graphs
 - Students will be able to analyze polar functions.

ASSESSMENTS

Pre-Assessment(s)	<ul style="list-style-type: none"> ● Solving Right Triangles Entrance Ticket
Formative	<ul style="list-style-type: none"> ● Homework ● Exit Tickets ● AP Classroom Problems
Summative	<ul style="list-style-type: none"> ● Standards Mapped Quizzes ● Challenge Questions ● Ferris Wheel Lab ● Enveloping Trig Lab
Benchmark / Common	<ul style="list-style-type: none"> ● Quarter 3 Exam

COLLEGE BOARD AP PRECALCULUS LEARNING STANDARDS (CBAPPS)

Must include the standard # & verbiage

- 3.1.A: Construct graphs of periodic relationships based on verbal representations.
- 3.1.B: Describe key characteristics of a periodic function based on a verbal representation.
- 3.2: Determine the sine, cosine, and tangent of an angle using the unit circle.
- 3.3: Determine coordinates of points on a circle centered at the origin.
- 3.4: Construct representations of the sine and cosine functions using the unit circle.
- 3.5: Identify key characteristics of the sine and cosine functions.

Hillsborough Township Public Schools Curriculum Map
Course Title: AP Precalculus

- 3.6: Identify the amplitude, vertical shift, period, and phase shift of a sinusoidal function.
- 3.7: Construct sinusoidal function models of periodic phenomena.
- 3.8.A: Construct representations of the tangent function using the unit circle.
- 3.8.B: Describe key characteristics of the tangent function.
- 3.8.C: Describe additive and multiplicative transformations involving the tangent function.
- 3.9: Construct analytical and graphical representations of the inverse of the sine, cosine, and tangent functions over a restricted domain.
- 3.10: Solve equations and inequalities involving trigonometric functions.
- 3.11: Identify key characteristics of functions that involve quotients of the sine and cosine functions.
- 3.12.A: Rewrite trigonometric expressions in equivalent forms with the Pythagorean identity.
- 3.12.B: Rewrite trigonometric expressions in equivalent forms with sine and cosine sum identities.
- 3.12.C: Solve equations using equivalent analytic representations of trigonometric functions.
- 3.13: Determine the location of a point in the plane using both rectangular and polar coordinates.
- 3.14: Construct graphs of polar functions.
- 3.15: Describe characteristics of the graph of a polar function.
- 4.8.D.2: The Law of Sines and Law of Cosines can be used to determine side lengths and angle measures of triangles formed by vector addition.

INTERDISCIPLINARY CONNECTIONS

NEW JERSEY STUDENT LEARNING STANDARDS (NJSL)

Must include the standard # & verbiage

Comprehensive Health & Physical Education

- 2.1.12.EH.1: Recognize one's personal traits, strengths, and limitations and identify how to develop skills to support a healthy lifestyle.
- 2.1.12.EH.3: Describe strategies to appropriately respond to stressors in a variety of situations.

English Language Arts

- RST.11-12.4 - Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context.
- W.11-12.1. Write arguments to support claims in an analysis of substantive topics or texts, using valid reasoning and relevant and sufficient evidence.

Computer Science & Design Thinking

Hillsborough Township Public Schools Curriculum Map

Course Title: AP Precalculus

- 8.1.2.CS.1: Select and operate computing devices that perform a variety of tasks accurately and quickly based on user needs and preferences.
- 8.1.2.DA.1: Collect and present data, including climate change data, in various visual formats.
- 8.1.2.DA.3: Identify and describe patterns in data visualizations.
- 8.1.2.DA.4: Make predictions based on data using charts or graphs.
- 8.1.2.AP.4: Break down a task into a sequence of steps.
- 8.1.5.DA.5: Propose cause and effect relationships, predict outcomes, or communicate ideas using data.

Science

- HS-PS4-1: Use mathematical representations to support a claim regarding relationships among the frequency, wavelength, and speed of waves traveling in various media.

Visual & Performing Arts

- None.

World Languages

- None.

CAREER READINESS, LIFE LITERACIES, & KEY SKILLS CONNECTIONS

Must include the standard # & verbiage

9.1-Personal Financial Literacy

- None.

9.2-Career Awareness, Exploration, Preparation, and Training

- 9.4.12.CI.1: Demonstrate the ability to reflect, analyze, and use creative skills and ideas.
- 9.4.5.DC.4: Model safe, legal, and ethical behavior when using online or offline technology.

9.4-Life Literacies & Key Skills

- 9.4.12.TL.3: Analyze the effectiveness of the process and quality of collaborative environments.
- 9.4.12.CT.2: Explain the potential benefits of collaborating to enhance critical thinking and problem solving.
- 8.2.2.ED.2: Collaborate to solve a simple problem, or to illustrate how to build a product using the design process.

CAREERS ASSOCIATED WITH THIS UNIT

- Physicist, biologist, meteorologist, musician

Hillsborough Township Public Schools Curriculum Map
Course Title: AP Precalculus

DIVERSITY, EQUITY, & INCLUSION CONNECTIONS

Required in grades K-12 per [N.J.S.A. 18A:35-4:36a](#) & the Amistad Law [N.J.S.A. 18A 52:16A-88](#)

Required in grades 7-12 per [N.J.S.A. 18A:35-4.35](#)

- None.

SOCIAL EMOTIONAL LEARNING CONNECTIONS

[NJ SEL](#) sub-competencies are listed that are addressed in this unit

Self-Awareness

- Recognize one's personal traits, strengths, and limitations
- Recognize the importance of self-confidence in handling daily tasks and challenges

Self-Management

- Understand and practice strategies for managing one's own emotions, thoughts, and behaviors
- Recognize the skills needed to establish and achieve personal and educational goals
- Identify and apply ways to persevere or overcome barriers through alternative methods to achieve one's goals

Social Awareness

- Recognize and identify the thoughts, feelings, and perspectives of others
- Demonstrate an awareness of the expectations for social interactions in a variety of settings

Relationship Skills

- Utilize positive communication and social skills to interact effectively with others
- Identify who, when, where, or how to seek help for oneself or others when needed

Responsible Decision-Making

- Develop, implement, and model effective problem-solving and critical thinking skills
- Identify the consequences associated with one's actions in order to make constructive choices

MODIFICATIONS/ACCOMMODATIONS - ELL, Special Education, Gifted, At Risk of Failure, 504

- Accommodations for all subject areas may be viewed [here](#).

RESOURCES – [Cited](#) print and electronic sources

- AP Classroom
- College Board
- Calc Medic
- Desmos

Hillsborough Township Public Schools Curriculum Map
Course Title: AP Precalculus

- GeoGebra
- Classpad.net (Casio)
- TI Education (Texas Instruments)

Hillsborough Township Public Schools Curriculum Map
Course Title: AP Precalculus

UNIT OF STUDY	Unit 4: Functions Involving Parameters, Vectors, and Matrices
PACING	51 days
ESSENTIAL QUESTIONS	
<ul style="list-style-type: none">● How can we determine when the populations of species in an ecosystem will be relatively steady?● How can we analyze the vertical and horizontal aspects of motion independently?● How does high resolution computer-generated imaging achieve smooth and realistic motion on screen with so many pixels?	
ENDURING UNDERSTANDINGS	
<ul style="list-style-type: none">● Parametric functions have multiple dependent variables' values paired with a single input variable or parameter so modeling scenarios allow students to explore change in terms of components.● Vectors can be broken into vertical and horizontal aspects using component form so they can be added or utilized together to determine, for example, the end force upon an object.● Matrices can be used to map a set of input vectors to output vectors which is the basis for vector-based computer graphics. Matrices implement a rotation on a set of vectors or a dilation on a set of vectors to help show their favorite video game character tripping and falling.	
LEARNING TARGETS	
<ul style="list-style-type: none">● Chapter 6: Vectors, Parametric Functions, and Conic Sections<ul style="list-style-type: none">○ 6.1 Vectors in the Plane<ul style="list-style-type: none">■ Students will be able to graph a vector and find its direction and magnitude when given the initial and terminal points.■ Students will be able to add vectors and find the product of a scalar k and a vector in component form.■ Students will be able to graph unit vectors.■ Students will be able to find the components of a vector when given a direction and magnitude.■ Students will be able to find the dot product of two vectors and use the properties of dot products.■ Students will be able to find the angle between two vectors and determine magnitude.■ Students will be able to apply vectors to force applications.○ 6.2 Parametric and Implicit Functions<ul style="list-style-type: none">■ Students will be able to graph parametric equations.■ Students will be able to eliminate a parameter to identify the graph.■ Students will be able to use parametric equations to analyze applications.	

Hillsborough Township Public Schools Curriculum Map

Course Title: AP Precalculus

- 6.3 Conic Sections and a New Look at Parabolas
 - Students will be able to find the focus, directrix, and focal width of a parabola.
 - Students will be able to find the equation of a parabola.
 - Students will be able to use standard form of a parabola to graph and analyze the relation.
- 6.4 Circles and Ellipses
 - Students will be able to define a circle parametrically.
 - Students will be able to find the center, foci, major axis, and minor axis of an ellipse.
 - Students will be able to find the equation of an ellipse.
 - Students will be able to use standard form of an ellipse to graph and analyze the relation.
 - Students will be able to define an ellipse parametrically.
- 6.5 Hyperbolas
 - Students will be able to find the center, vertices, foci, transverse axis, conjugate axis, and asymptotes of a hyperbola.
 - Students will be able to find the equation of a hyperbola.
 - Students will be able to use standard form of a hyperbola to graph and analyze the relation.
 - Students will be able to define a hyperbola parametrically.
- Chapter 7: Linear Transformations and Matrix-Vector Functions
 - 7.1 Matrix Algebra
 - Students will be able to determine the order of a matrix.
 - Students will be able to add and subtract matrices.
 - Students will be able to complete scalar multiplication on a matrix.
 - Students will be able to multiply matrices.
 - Students will be familiar with an identity matrix.
 - Students will be able to find the determinant and inverse of a 2x2 matrix.
 - 7.2 Linear Transformations and Matrices
 - Students will be able to explore linear transformations with matrices.
 - 7.3 Linear Transformations of the xy-plane
 - Students will be able to explore matrix multiplication on vectors.
 - Students will be able to explore rotations, dilations, and reflections in the x-y plane.
 - 7.4 Transitions Matrices and Modeling
 - Students will be able to explore modeling using matrices.
- Chapter 8: Limits, Derivatives, and Integrals
 - 8.3 More on Limits
 - Students will be able to define a limit.
 - Students will be able to use properties of limits including sum, difference, product, constant multiple, quotient, power, and root rule to evaluate.
 - Students will be able to find limits of a continuous function.

Hillsborough Township Public Schools Curriculum Map

Course Title: AP Precalculus

- Students will be able to find one and two sided limits.
- Students will be able to find limits as it approaches infinity using tables and graphs.

ASSESSMENTS

Pre-Assessment(s)	<ul style="list-style-type: none"> ● None
Formative	<ul style="list-style-type: none"> ● Homework ● Exit Tickets ● AP Classroom Problems ● AP Practice Test
Summative	<ul style="list-style-type: none"> ● Standards Mapped Quizzes ● Challenge Questions ● Limit Mind Map
Benchmark / Common	<ul style="list-style-type: none"> ● Final Exam

COLLEGE BOARD AP PRECALCULUS LEARNING STANDARDS (CBAPPS)

Must include the standard # & verbiage

- 4.1: Construct a graph or table of values for a parametric function represented analytically.
- 4.2: Identify key characteristics of a parametric planar motion function that are related to position.
- 4.3: Identify key characteristics of a parametric planar motion function that are related to direction and rate of change.
- 4.4: Express motion around a circle or along a line segment parametrically.
- 4.5.A: Construct a graph of an equation involving two variables.
- 4.5.B: Determine how the two quantities related in an implicitly defined function vary together.
- 4.6: Represent conic sections with horizontal or vertical symmetry analytically.
- 4.7.A: Represent a curve in the plane parametrically.
- 4.7.B: Represent conic sections parametrically.
- 4.8.A: Identify characteristics of a vector.
- 4.8.B: Determine sums and products involving vectors.
- 4.8.C: Determine a unit vector for a given vector.
- 4.8.D: Determine angle measures between vectors and magnitudes of vectors involved in vector addition.
- 4.9: Represent planar motion in terms of vector-valued functions.
- 4.10: Determine the product of two matrices.
- 4.11.A: Determine the inverse of a 2×2 matrix.
- 4.11.B: Apply the value of the determinant to invertibility and vectors.
- 4.12: Determine the output vectors of a linear transformation using a 2×2 matrix.
- 4.13.A: Determine the association between a linear transformation and a matrix.
- 4.13.B: Determine the composition of two linear transformations.

Hillsborough Township Public Schools Curriculum Map

Course Title: AP Precalculus

- 4.13.C: Determine the inverse of a linear transformation.
- 4.14.A: Construct a model of a scenario involving transitions between two states using matrices.
- 4.14.B: Apply matrix models to predict future and past states for n transition steps.

INTERDISCIPLINARY CONNECTIONS

NEW JERSEY STUDENT LEARNING STANDARDS (NJSL)

Must include the standard # & verbiage

Comprehensive Health & Physical Education

- 2.1.12.EH.1: Recognize one's personal traits, strengths, and limitations and identify how to develop skills to support a healthy lifestyle.
- 2.1.12.EH.3: Describe strategies to appropriately respond to stressors in a variety of situations.

English Language Arts

- RST.11-12.4 - Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context.
- W.11-12.1. Write arguments to support claims in an analysis of substantive topics or texts, using valid reasoning and relevant and sufficient evidence.

Computer Science & Design Thinking

- 8.1.2.CS.1: Select and operate computing devices that perform a variety of tasks accurately and quickly based on user needs and preferences.
- 8.1.2.DA.1: Collect and present data, including climate change data, in various visual formats.
- 8.1.2.DA.3: Identify and describe patterns in data visualizations.
- 8.1.2.DA.4: Make predictions based on data using charts or graphs.
- 8.1.2.AP.4: Break down a task into a sequence of steps.
- 8.1.5.DA.5: Propose cause and effect relationships, predict outcomes, or communicate ideas using data.

Science

- HS-ESS1-4: Use mathematical or computational representations to predict the motion of orbiting objects in the solar system.

Visual & Performing Arts

- None

World Languages

- None.

Hillsborough Township Public Schools Curriculum Map
Course Title: AP Precalculus

CAREER READINESS, LIFE LITERACIES, & KEY SKILLS CONNECTIONS

Must include the standard # & verbiage

9.1-Personal Financial Literacy

- None.

9.2-Career Awareness, Exploration, Preparation, and Training

- 9.4.12.Cl.1: Demonstrate the ability to reflect, analyze, and use creative skills and ideas.
- 9.4.5.DC.4: Model safe, legal, and ethical behavior when using online or offline technology.

9.4-Life Literacies & Key Skills

- 9.4.12.TL.3: Analyze the effectiveness of the process and quality of collaborative environments.
- 9.4.12.CT.2: Explain the potential benefits of collaborating to enhance critical thinking and problem solving.
- 8.2.2.ED.2: Collaborate to solve a simple problem, or to illustrate how to build a product using the design process.

CAREERS ASSOCIATED WITH THIS UNIT

- Data analyst, astronomer

DIVERSITY, EQUITY, & INCLUSION CONNECTIONS

Required in grades K-12 per [N.J.S.A. 18A:35-4:36a](#) & the Amistad Law [N.J.S.A. 18A 52:16A-88](#)

Required in grades 7-12 per [N.J.S.A. 18A:35-4.35](#)

- None.

SOCIAL EMOTIONAL LEARNING CONNECTIONS

***NJ SEL** sub-competencies are listed that are addressed in this unit*

Self-Awareness

- Recognize one's personal traits, strengths, and limitations
- Recognize the importance of self-confidence in handling daily tasks and challenges

Self-Management

- Understand and practice strategies for managing one's own emotions, thoughts, and behaviors
- Recognize the skills needed to establish and achieve personal and educational goals
- Identify and apply ways to persevere or overcome barriers through alternative methods to achieve one's goals

Hillsborough Township Public Schools Curriculum Map
Course Title: AP Precalculus

Social Awareness

- Recognize and identify the thoughts, feelings, and perspectives of others
- Demonstrate an awareness of the expectations for social interactions in a variety of settings

Relationship Skills

- Utilize positive communication and social skills to interact effectively with others
- Identify who, when, where, or how to seek help for oneself or others when needed

Responsible Decision-Making

- Develop, implement, and model effective problem-solving and critical thinking skills
- Identify the consequences associated with one's actions in order to make constructive choices

MODIFICATIONS/ACCOMMODATIONS - *ELL, Special Education, Gifted, At Risk of Failure, 504*

- Accommodations for all subject areas may be viewed [here](#).

RESOURCES – *Cited print and electronic sources*

- AP Classroom
- College Board
- Calc Medic
- Desmos
- GeoGebra
- Classpad.net (Casio)
- TI Education (Texas Instruments)